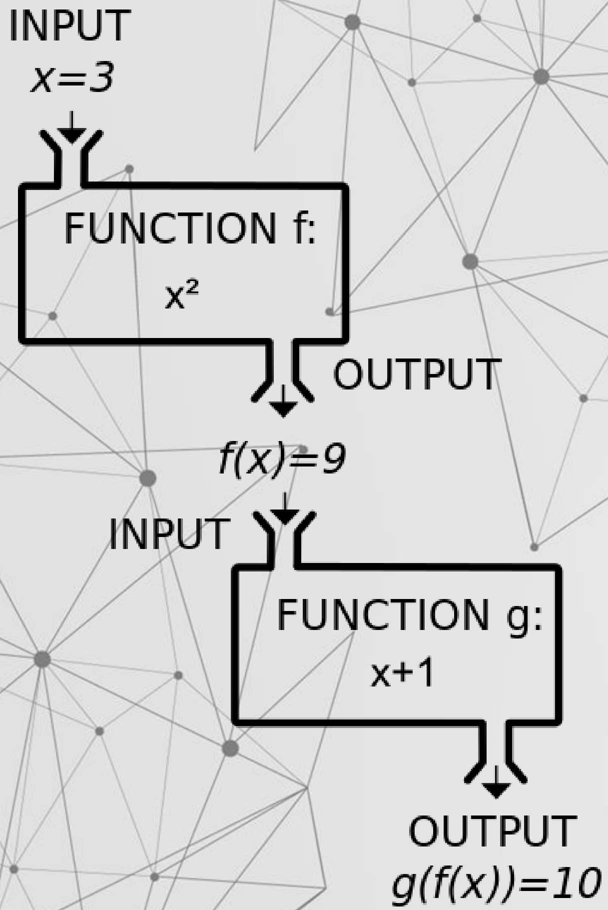
# CALCULUS 1

Prihandoko Rudi




# 02
# FUNCTION

Definition, Notation, and Operation

# Relation

## Product of Two Sets

Let $A$ and $B$ be two sets. The product of $A$ and $B$ is a set that contains all of ordered pairs $(a, b)$ for which $a$ is in $A$ and $b$ is in $B$. We denote the product as follows.

$$A \times B = \{(a, b) \mid a \in A, b \in B\}$$

## Definition

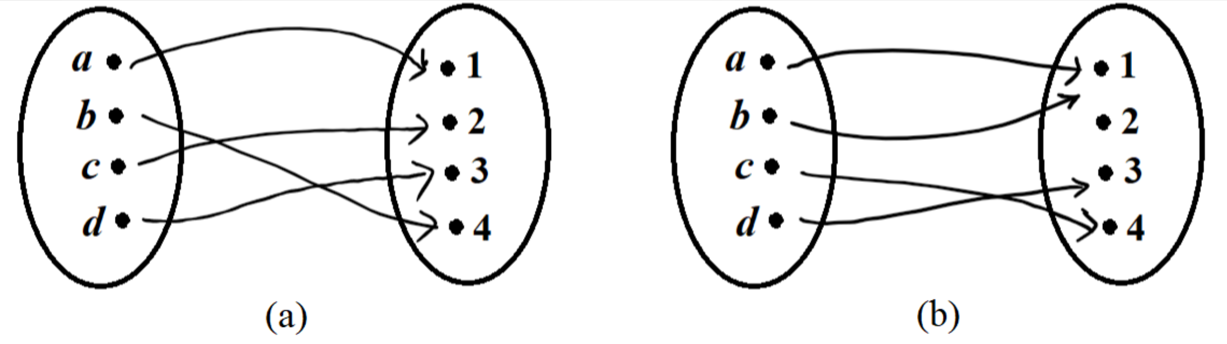
Let $A$ and $B$ be two sets. A **relation** from $A$ to $B$ is a subset of $A \times B$.

# Function

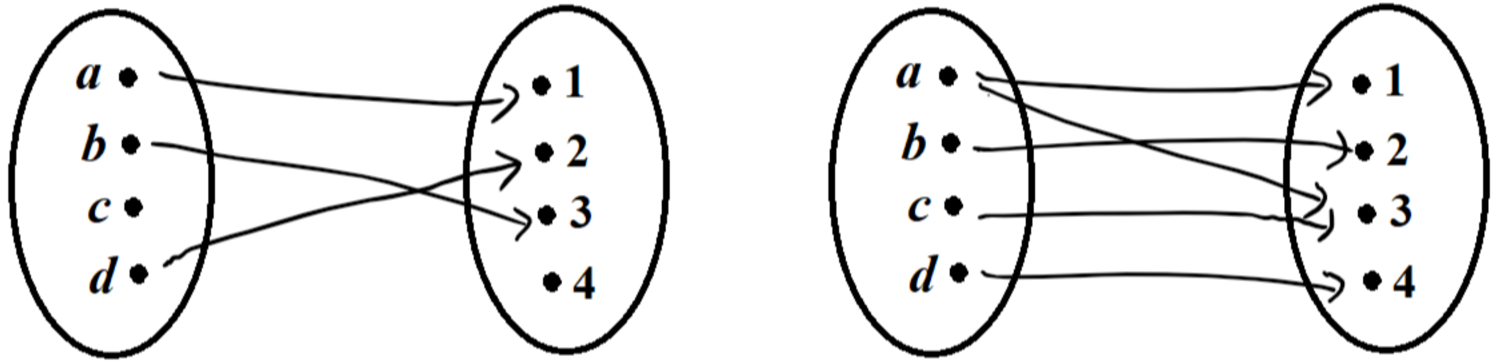
## Definition

Let $A$ and $B$ be two sets. A **function** from $A$ to $B$ is a relation that associates each element of the set $A$, to a single element of the set $B$.

In this case, we said that $A$ is the **domain** and $B$ is the **codomain**. The set of all element of codomain that associated with an element (one or more) of the domain called the **range** of the function.



# Definition



(a) (b)

# Notation (1)

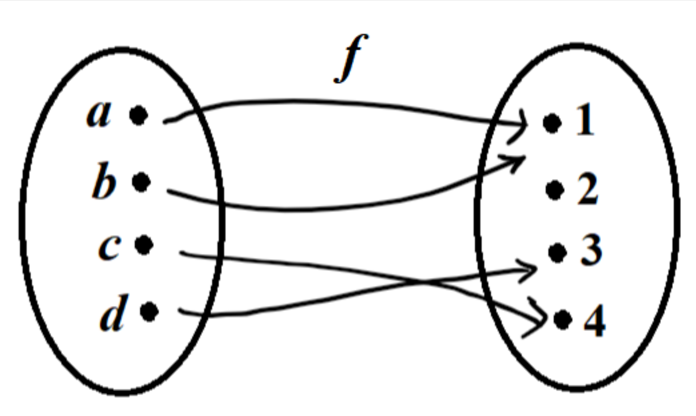
- $f, g, h, a, b, A, B, f_2, g_3$, etc.
- $\alpha, \beta, \gamma, \sigma, \mu, \alpha_f, \beta_2$, etc.
- $|\cdot|$, $\lfloor \cdot \rfloor$, $< \cdot, \cdot >$,
- if $A$ is the domain and $B$ is the codomain $B$ we also write function form

$$f : A \to B$$

or

$$x : A \mapsto B$$

# Notation (2)




If the function are as shown, we may also denote the function in the form

$$\{f(a) = 1, f(b) = 1, f(c) = 4, f(d) = 3\}.$$

# Notation (3)

Let $f : A \to B$ is a function.

- The domain $A$ of $f$ notated by $D_f$.
- The range of $f$ notated by $R_f$. We may also write

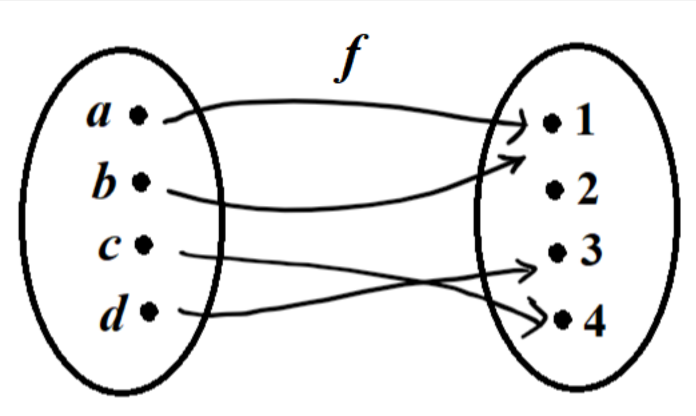
$$R_f = \{y \in B \mid y = f(x) \text{ for some } x\}.$$

# Remarks

## Remarks

If the domain of a function is not given explicitely, may refer to the biggest subset of reals that the function is well defined.

# Introduction to Graphic Function

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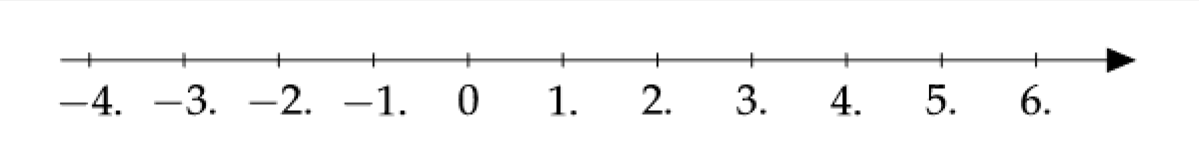
$$\{f(a) = 1, f(b) = 1, f(c) = 4, f(d) = 3\}.$$

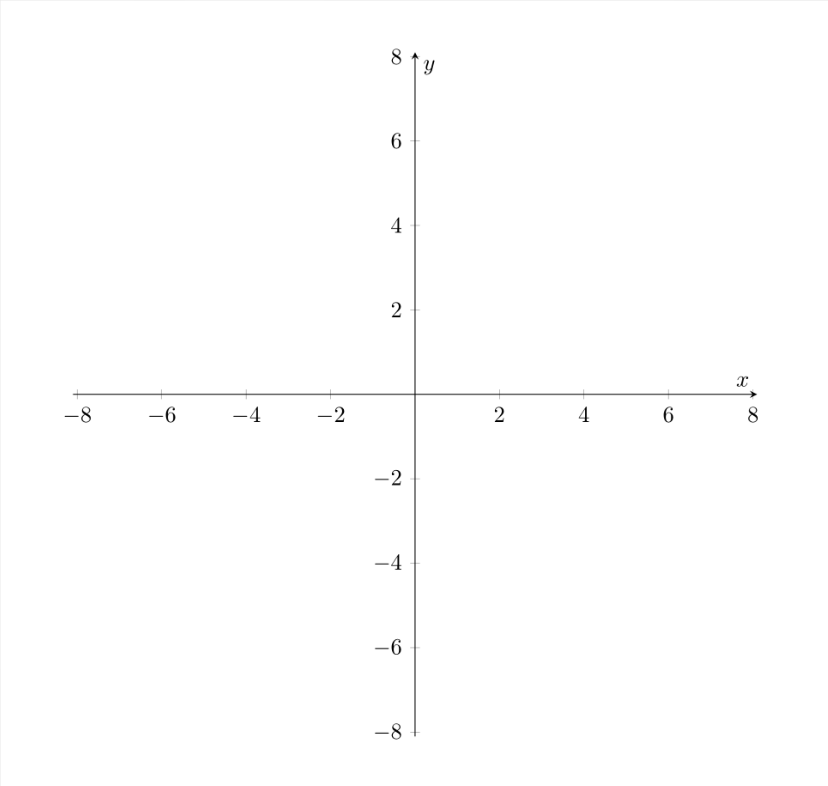
# Introduction to Graphic Function

The set of real numbers is completely ordered.
Hence, it can be represented as a line.

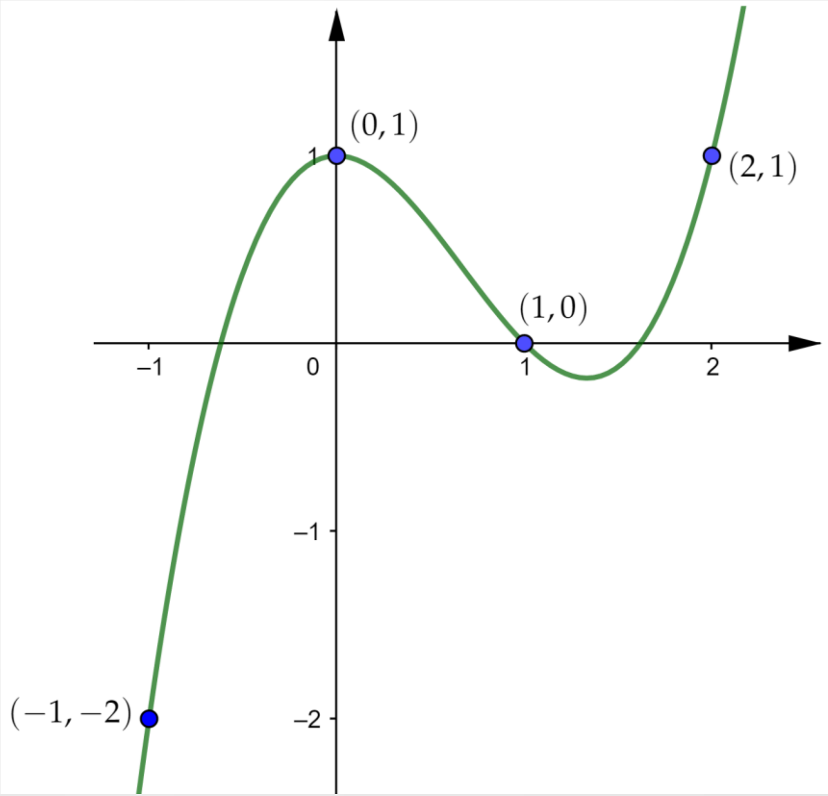



The set of real numbers is completely ordered.
How about pairs?

# Introduction to Graphic Function



# Introduction to Graphic Function



# Example

**Example**

- A function $f(x) = 3x - 2$ has $\mathbb{R}$ as the domain.
  All polinomial function has $\mathbb{R}$ as the domain.

- The domain of the function $g(x) = \sqrt{x}$ is non-negative real numbers.
  The domain of the function $g_2(x) = \sqrt{x-2}$ is all real numbers for which $(x-2)$ is positive.

- The domain of the function $h(x) = \frac{1}{x}$ is non-zero real numbers.
  The domain of $h_3(x) = \frac{1}{x-3}$ is all real numbers except 3.

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- The logarithm function log has domain of positive real numbers.

# 02
# FUNCTION

Some special function

# Injective, Surjective, Bijective

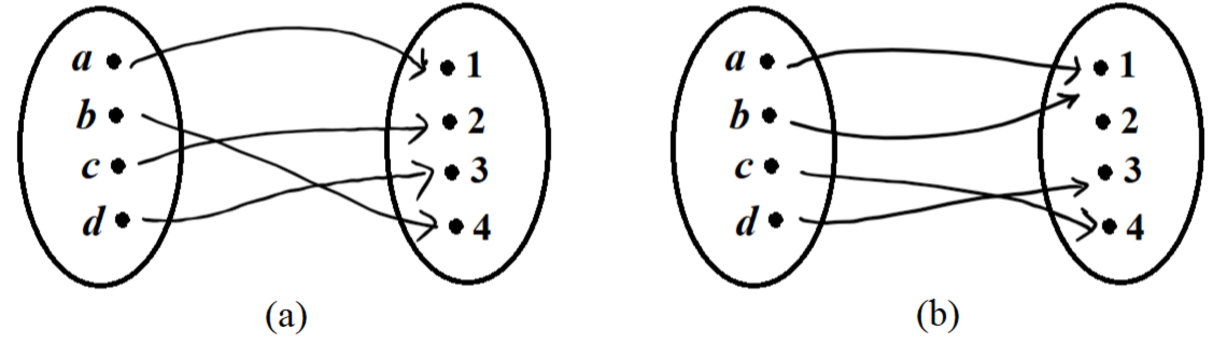
## Definition

A function is called **injective** function if for any different elements on the domain has differents images.

A function is called **surjective** function if for any different elements on the range has differents pre-images.

A function is called **bijective** function if its both injective and surjective.

# Injective, Surjective, Bijective



# Inverse Function

## Definition

Given function $f : A \to B$. An **inverse** of $f$ is a relation from $B$ to $A$ which associates an element of $B$ with its pre-image (with respect to $f$) if any.

If the inverse of function $f$ is also a function, then we called it **the inverse function** of $f$ and denotes $f^{-1}$.

# Example

## Example

- A function $f(x) = 3x - 2$ has inverse $f^{-1}(x) = \frac{1}{3}(x+2)$.
  The domain of $f^{-1}$ are all reals.

- The function $g(x) = \sqrt{x}$ has inverse function $f^{-1}(x) = x^2$.
  But, $g(x) = x^2$ does not have inverse function.

- The function $h(x) = \frac{1}{x}$ has inverse $f(x) = \frac{1}{x}$ that is itself.
  The function $h_3(x) = \frac{1}{x-3}$ has inverse $f(x) = \frac{1}{x} + 3$.

- The logarithm function log has inverse $f(x) = 10^x$ with domain all reals.

# Finding Inverse

## Outline

- Write the function as $f(x) = y$.
- With some manipulation, try to make the expression $x$ is only appears on one hand.
- Let's see whether its function.
- Try to figure out what is the natural domain.
- The logarithm function log has inverse $f(x) = 10^x$ with domain all reals.

# Composition of Functions

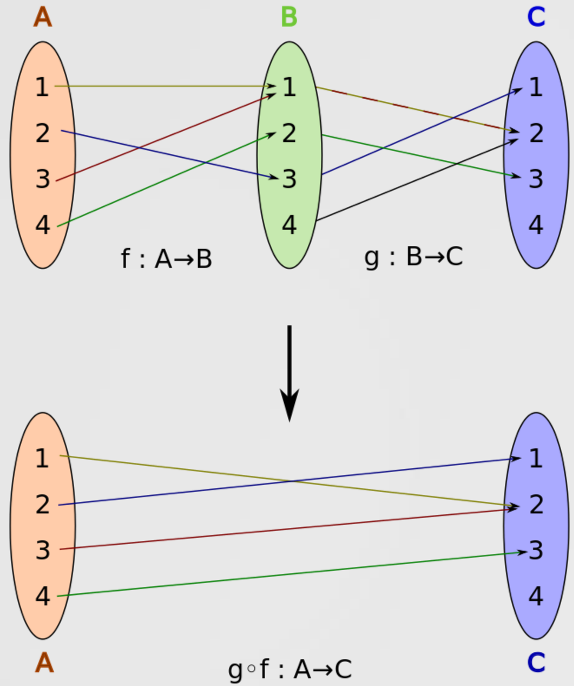
## Definition

Given function $f : A \to B$ and function $g : B \to C$ such that $D_g \subset R_f$. The composition of function $g \circ f$ defined as follows.

$$g \circ f(x) = g(f(x))$$

The domain of function $f \circ g$ is $D_{f \circ g} = \{x \in D_f \mid f(x) \in D_g\}$.

# Composition of Functions



# Example

## Example

Given function $f(x) = 3x - 2$, $g(x) = \sqrt{x}$, and $h(x) = \frac{1}{x}$.
We will try to find $g \circ f$ and $f \circ g$. Let $y = f(x)$.

$$g \circ f = g(f(x)) = g(y) = \sqrt{y} = \sqrt{3x - 2}$$

Its domain should fulfill $(3x - 2) > 0$ or equivalently $x > \frac{2}{3}$. The domain $D_f$ is

$$D_f = \{x \in \mathbb{R} \mid x > 2\}$$

# Finding Composition

## Outline

Given function $f$, $g$, and $g \circ f$.

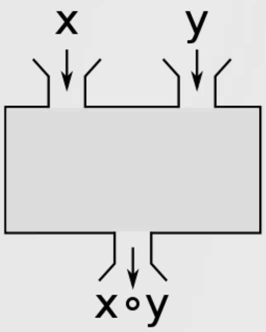
- Write the function as $f(x) = y$.
- Write $g \circ f(x) = g(f(x)) = g(y)$
- Substitute $y$ with actual function $y = \dots$
- Find the domain $g \circ f$

# Multivariate Functions

## Definition

A **multivariate function**, or function of several variables is a function that depends on several arguments.

Such functions are commonly encountered. For example, the position of a car on a road is a function of the time and its speed.



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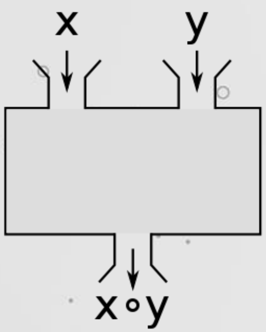

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## Definition

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Given $f$ is function defined on $A$ and $g$ is a function defined on $B$ and $A \cap B =$. A piecewise function $h$ on $A \cup B$ is a defined with $f$ on $A$ and $g$ on $B$ notated as follows.

$$h(x) = \begin{cases} f(x), & \text{if } x \in A \\ g(x), & \text{if } x \in B. \end{cases}$$

# Piecewise Functions: Example

**The absolute value**

$$h(x) = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0. \end{cases}$$

**Rectified Linear Unit (ReLU)**

$$h(x) = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0. \end{cases}$$

# 02

# FUNCTION

Function on Real Numbers

# Constant and identity function

The **identity function** on real numbers is the function $f(x) = x$ for all real numbers $x$. A **constant function** is the function $f(x) = c$ for some fix $c$, for all real numbers $x$.

# Algebraic Function

A function that is a root of a nonzero polynomials equation.

# Algebraic Function

A function that consist of addition, substraction, multiplication, division, and/or  root operation of constant function and identity function.

# Transcendental Number

A function that is not algebraic

# Algebraic Function

- Polynomials function $P(x) = a_n x^n + a_{n-1}^{n-1} + \cdots + a_1 x + a_0$ for some reals $a_i$, $i = 1, 2, 3, ..., n$, and $a_n \neq 0$. The **degree** of the polynomials is the coefficient of the highest order of $x$, that is $a_n$.
- Rationals function $R(x) = P(x)/Q(x)$ which $P$ and $Q$ are both polynomials.
- Root function $\sqrt{x}$.
- Absolute value function $|x|$.
- and permutation of the above.

# Trancendental Function

- Exponentials function, $f(x) = e^x, g(x) = 2^x, H(x) = 1.001^x$
- Logarithmic function or inverse of exponential function
- Trigonometric function, $\sin, \cos, \tan, \cot, \sec, \csc$
- Cyclometric function or inverse of trigonometric function

# THANKS

Does anyone have any questions?

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