Calculus 1

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CALCULUS

Calculus, originally called **infinitesimal calculus** or "the calculus of infinitesimals", is the mathematical study of continuous change, in the same way that geometry is the study of shape and algebra is the study of generalizations of arithmetic operations.

--- Wikipedia

WELCOME!

Guillaume de l'Hôpital

Paris, France
2 February 1704

Gottfried Wilhelm Leibniz

Leipzig, Electorate of Saxony, Holy Roman Empire
1 July 1646

Brook Taylor

Durham, England
29 December 1731

Colin Maclaurin

Argyll, Scotland
February 1698

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Preliminaries

Notation and Terms

SOME NOTATIONS

$$5x^3 + 7 = (x+1)(x+2)$$

SOME NOTATIONS

power

constant

coefficient

$$5x^3 + 7 = (x + 1)(x + 2)$$

terms terms factors factors

SOME NOTATIONS

A set is a group of objects that obey some rules.

SOME NOTATIONS

A set is a group of objects that obey some rules.

$$A = \{x \mid some\ properties\}$$

A is (some description or properties).

SOME NOTATIONS

An element of a set, if its has the properties.

If x is an element of A, we write it $x \in A$.

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Real Numbers

Introduction

NUMBERS

NATURAL NUMBERS

1,2,3,...

$\mathbb{N}$

INTEGERS

...,-3,-2,-1,0,1,2,3,...

$\mathbb{Z}$

RATIONAL NUMBERS

a/b,   b≠0

$\mathbb{Q}$

REAL NUMBERS

Completion of rational number

$\mathbb{R}$

INFINITY?

What is infinity?

∞

COMPLEX NUMBERS?

Have a solutions for $x^2=-1$

$\mathbb{C}$

Algebraic Number

A number that is a root of a nonzero polynomials equation with integer or rational coefficient.

Transcendental Number

A number that is not algebraic

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Real Numbers

Operations and Properties

Operations and Properties

Addition

commutative

$$a + b = b + a$$

associative

$$(a + b) + c = a + (b + c)$$

identity elements

$$a + 0 = a$$

inverse

$$a + (-a) = 0$$

Multiplication

commutative

$$a \times b = b \times a$$

associative

$$(a \times b) \times c = a \times (b \times c)$$

identity elements

$$a \times 1 = a$$

inverse

$$a \times \frac{1}{a} = 1$$

Connected

distributive

$$a(b + c) = ab + ac$$

Operations and Properties

Regarding Minus

$$a \times b = ab$$

$$a \times (-b) = -ab$$

$$(-a) \times b = -ab$$

$$(-a) \times (-b) = ab$$

$$-(-a) = a$$

Operations and Properties

Regarding Zero

$$a \times 0 = 0$$

$$0 \times a = 0$$

If $ab = 0$ then it must be $a = 0$ or $b = 0$.

One may not divide by zero.

Operations and Properties

The operations on real numbers are obey the following hierarcy.

1. Parenthesis
2. (not necessarily following by) brackets, curly braces
3. power
4. multiplication, division
5. addition, substraction

Operations and Properties

The Hard Way to Compute

$$\frac{a}{b} + \frac{c}{d} =$$

$$\frac{a}{c} \times \frac{b}{d} =$$

Quiz

Compute

$$9 + 9 \times 9 - 9 \div 9 \times 0 =$$

Quiz

True or False

$$6\frac{5}{3} = 10$$

Quiz

True or False

$$-1^2 = 1$$

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Real Numbers

Order Relation

Order Relation

Any real numbers a satisfies exactly one of the following conditions:

- $a > 0$
- $a < 0$
- $a = 0$

Order Relation

Definition

Let a and b be real numbers.

- $a > b$ if $a - b > 0$
- $a < b$ if $a - b < 0$
- $a \leq b$ if $a < b$ or $a = b$
- $a \geq b$ if $a > b$ or $a = b$

Order Relation

Corollary

Let a, b, and c be real numbers.

- if $a \leq b$, then $a + c \leq b + c$
- if $a \leq b$ and $b \leq c$, then $a \leq c$
- if $a \leq b$ and $c > 0$, then $ac \leq bc$
- if $a \leq b$ and $c < 0$, then $ac \geq bc$
- $a^2 \geq 0$
- if $a > 0$ then $\frac{1}{a} > 0$
- if $0 < a \leq b$ then $0 < \frac{1}{b} \leq \frac{1}{a}$
- if $a, b > 0$ then

  $$a^2 \leq b^2 \iff a \leq b \iff \sqrt{a} \leq \sqrt{b}$$

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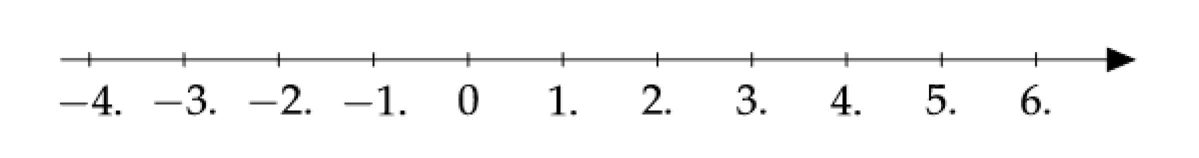
Real Numbers

Real lines and Intervals

Real lines and Intervals

Since any real number can be compared, then they are "completely ordered".



Real lines and Intervals

Type of intevals

Let a and b be real numbers such that $a < b$.
Then we have following type of intervals.

- closed bounded $[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\}$
- open bounded $(a, b) = \{x \in \mathbb{R} \mid a < x < b\}$
- bounded*
  - $(a, b] = \{x \in \mathbb{R} \mid a < x \leq b\}$
  - $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$

Real lines and Intervals

Type of intevals

Let a and b be real numbers such that $a < b$.
Then we have following type of intervals.

- unbounded
  - $[a, \infty) = \{x \in \mathbb{R} \mid a \leq x\}$
  - $(a, \infty) = \{x \in \mathbb{R} \mid a < x\}$
  - $(\infty, b] = \{x \in \mathbb{R} \mid x \leq b\}$
  - $(\infty, b) = \{x \in \mathbb{R} \mid x < b\}$

Type of intevals

Let a and b be real numbers such that $a < b$.

bounded interval

unbounded interval

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Real Numbers

Coordinates Systems: Inequalities

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Real Numbers

Coordinates Systems: Absolute Values

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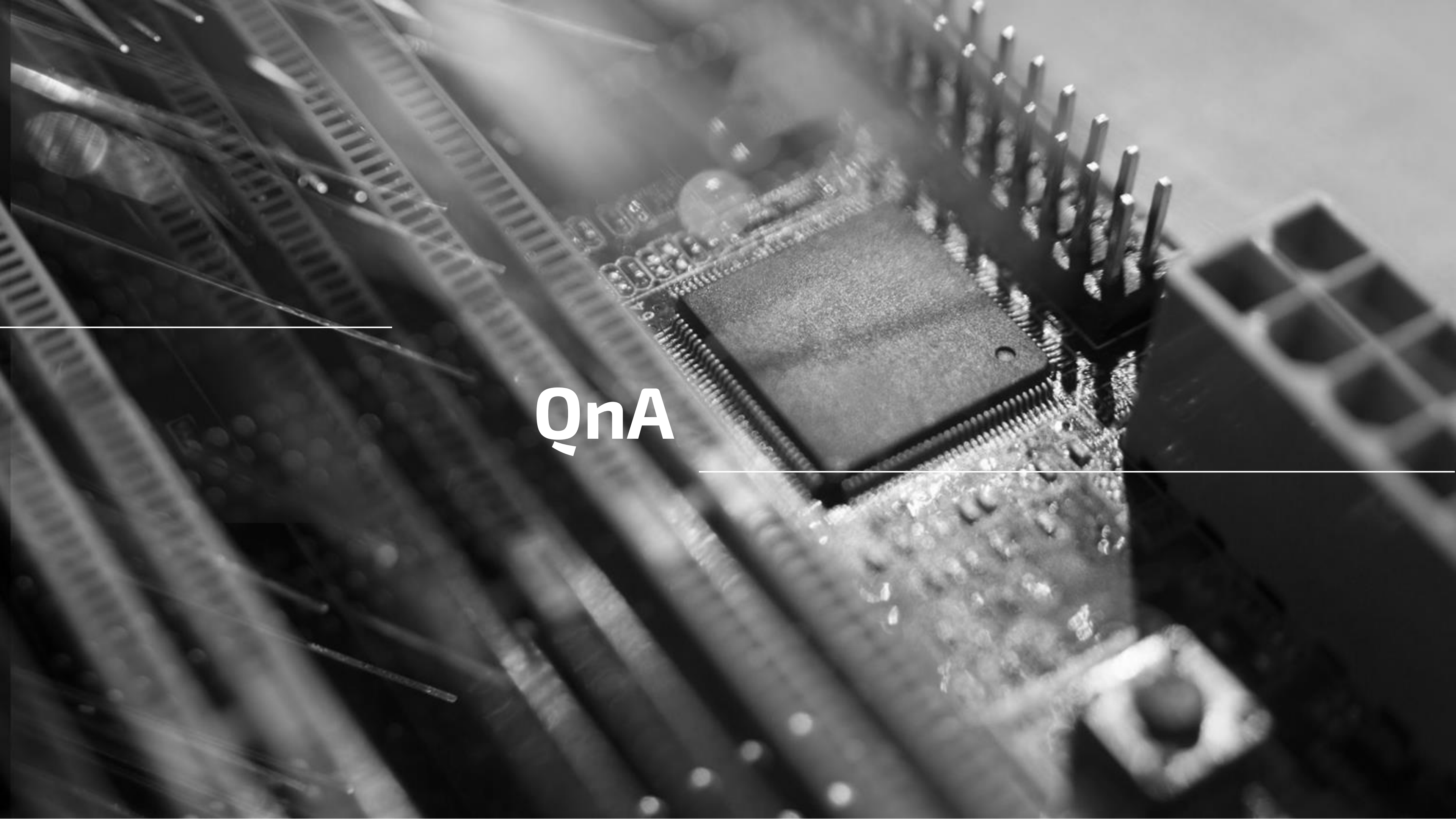
Real Numbers

Coordinates Systems: Cartesians and Polar



QnA

THANKS

Does anyone have any questions?

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