Limits

Calculus 1

Continuity

Limits


CONTINUE

Continue: Definition?

verb (used without object), **con·tin·ued, con·tin·u·ing.**

1 to go on after suspension or interruption:
The program continued after an intermission.

2 to go on or keep on, as in some course or action; extend:
The road continues for three miles.

3 to last or endure:
The strike continued for two months.

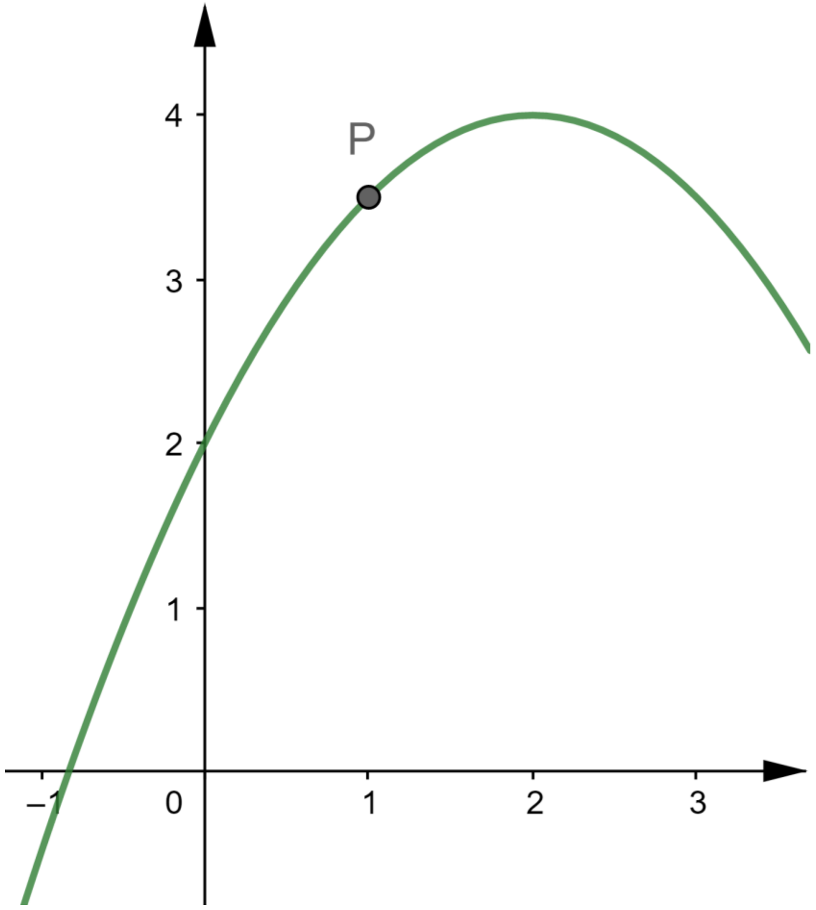
SEE MORE

verb (used with object), **con·tin·ued, con·tin·u·ing.**

6 to go on with or persist in:
to continue an action.

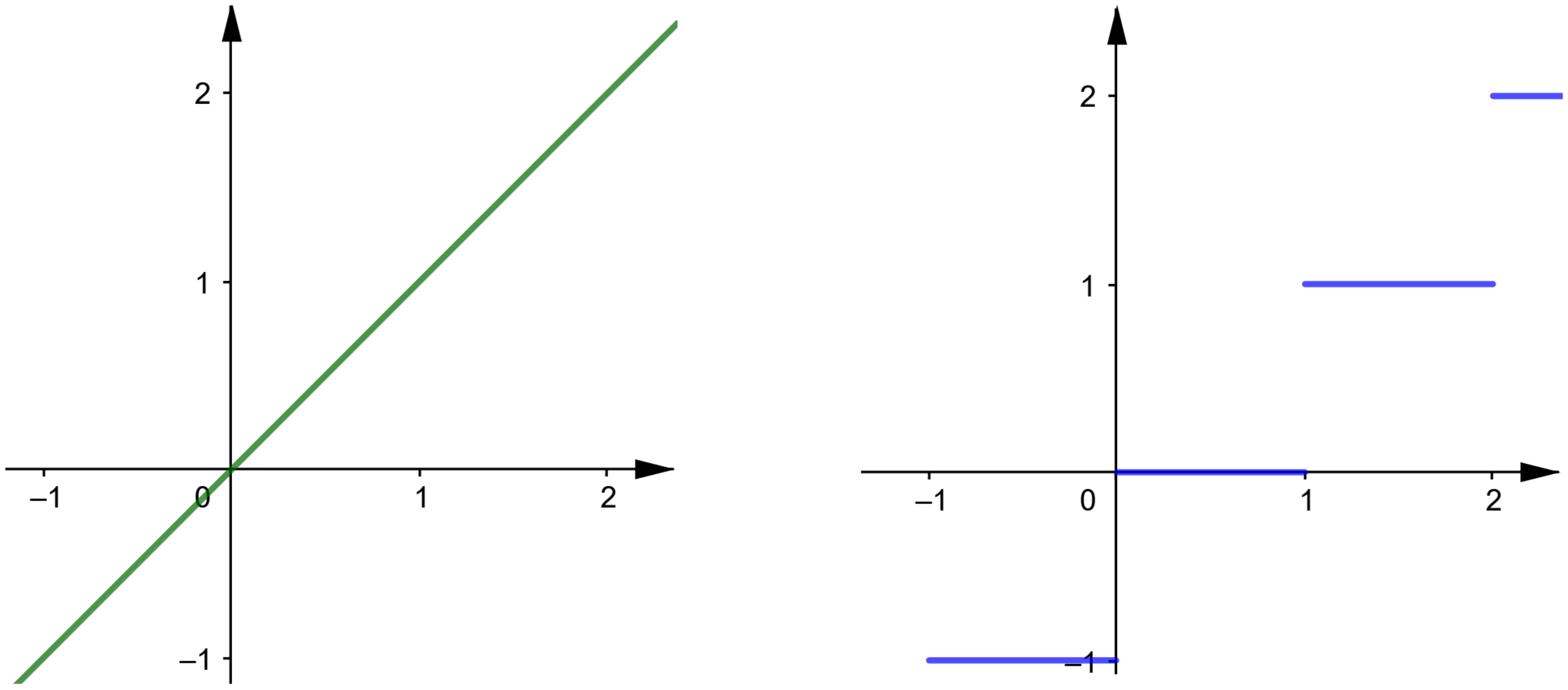
7 to carry on from the point of suspension or interruption:
He continued the concert after the latecomers were seated.

8 to extend from one point to another in space; prolong.



Continue: “Definition”

- Continue at a point $x = a$:
  - There isn’t any “jump” around $x = a$
- Continue on an interval:
  - The graphics can be traced with pencil without lift the pencils
  - The points on the graphics always “connected”



Continue: Why is it important?

- If function f continue at $x = a$, then the value $f(a)$ could be approximated with the value of the function in the neighbourhood.
  - Example: $f(\pi)$ approximated by the sequences
    $$f(3), f(3{,}1), f(3{,}14), f(3{,}141), f(3{,}1415), \ldots$$
- Equation $f(x) = 0$ could be numerically solved with some method such as bisection method
  - Example: the equation $x^2 - 2 = 0$
- The continuity of a function is a sufficient condition for the function to have a tangent line. .
  - Contoh: fungsi *floor* $f(x) = \lfloor x \rfloor$ tidak memiliki garis singgung di $x = 1$

Continuity at a point

Limit

Continuity at a point

Definition. A function f is continuous at a number a if

$$\lim_{n \to a} f(a)$$

Continuity at a point

Definition. A function f is *continuous* at a number a if

$$\lim_{x \to a} f(x) = f(a)$$

Requirements for this equation:

- the value $f(a)$ is defined;
- the limit $\lim_{x \to a} f(x)$ exists;
- the two are equal.

Continuity at a point

Definition. A function f is *continuous* at a number a if

$$\lim_{x \to a} f(x) = f(a)$$

Requirements for this equation:

- the value $f(a)$ is defined;
- the limit $\lim_{x \to a^+} f(x)$ exists;
- the limit $\lim_{x \to a^-} f(x)$ exists; and
- the three are equal.

Continuity at a point

Theorem. Let two functions f and g are continuous at some point a. Let k be a real number. Then we have the following functions are also continuous.

- $f + g$
- $f - g$
- fg
- kf and kg
- $\frac{f}{g}$ if $g(a) \neq 0$

Continuity at a point

Definition. A function f is ***discontinuous*** at a number a if it is not continuous at that point

Continuity at a point

Definition. A function f is <u>*discontinuous*</u> at a number a if it is not continuous at that point

Three types of discontinuity

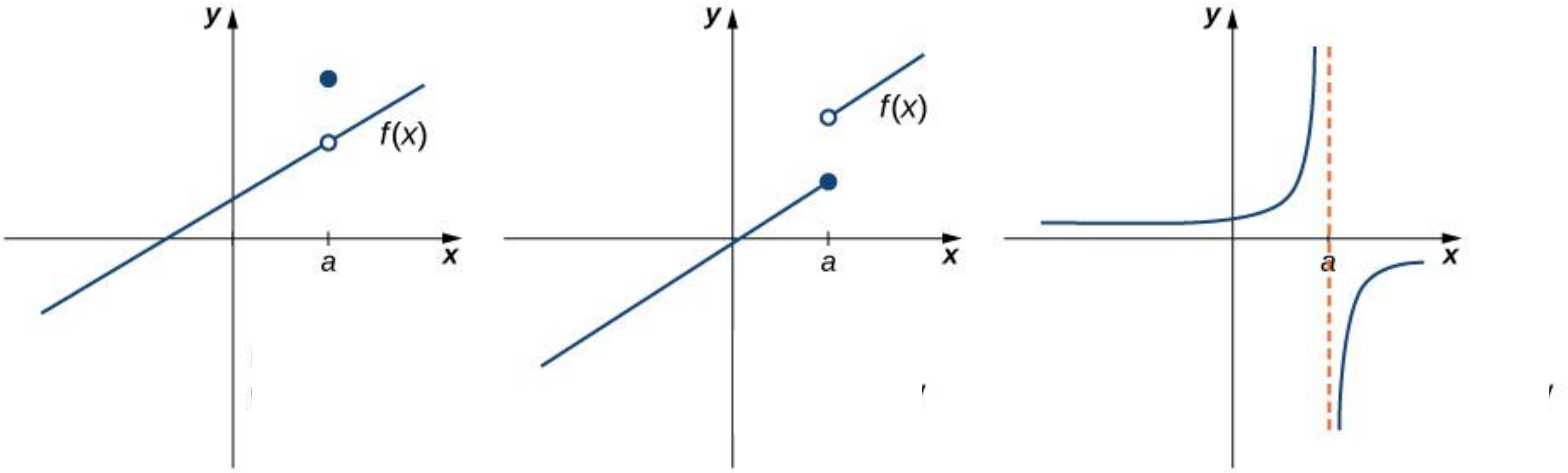
- Removable
- Jump
- Infinite

Continuity at a point

Three types of discontinuity

- Removable
  - If $\lim_{x \to a} f(x)$ exist
- Jump
  - If $\lim_{x \to a^-} f(x)$ and $\lim_{x \to a^+} f(x)$ exist but $\lim_{x \to a} f(x) \neq \lim_{x \to a} f(x)$
- Infinite
  - If $\lim_{x \to a^-} f(x) = \pm\infty$ or $\lim_{x \to a^+} f(x) = \pm\infty$ or both

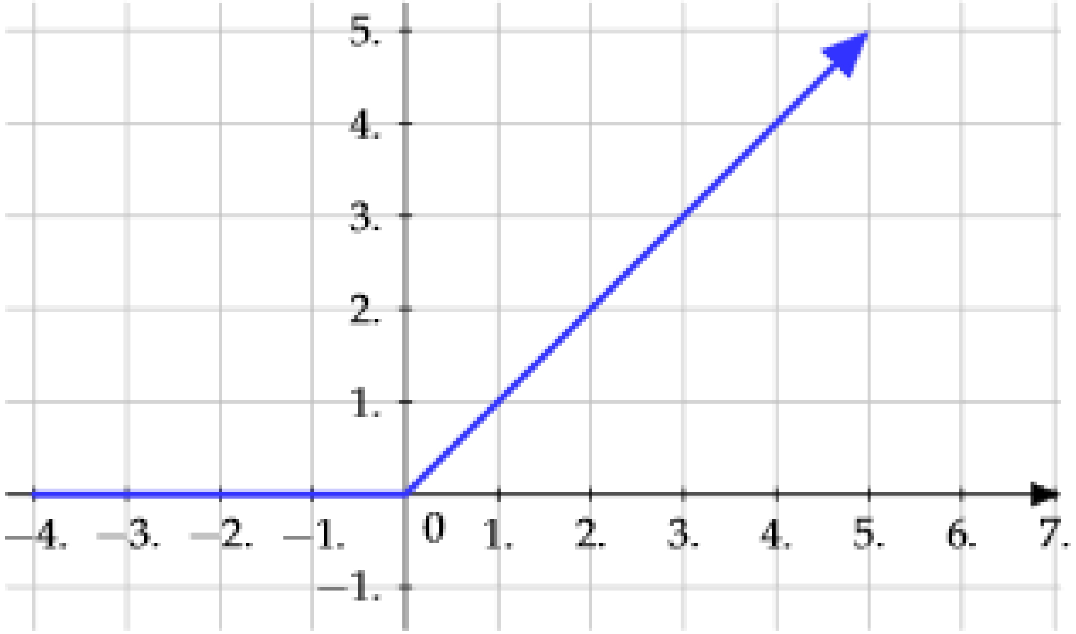
Type of Discontinuity



Example

- Consider ReLU function at $x = 0$.

$$f(x) = \begin{cases} x & x \geq 0 \\ 0 & x < 0 \end{cases}$$

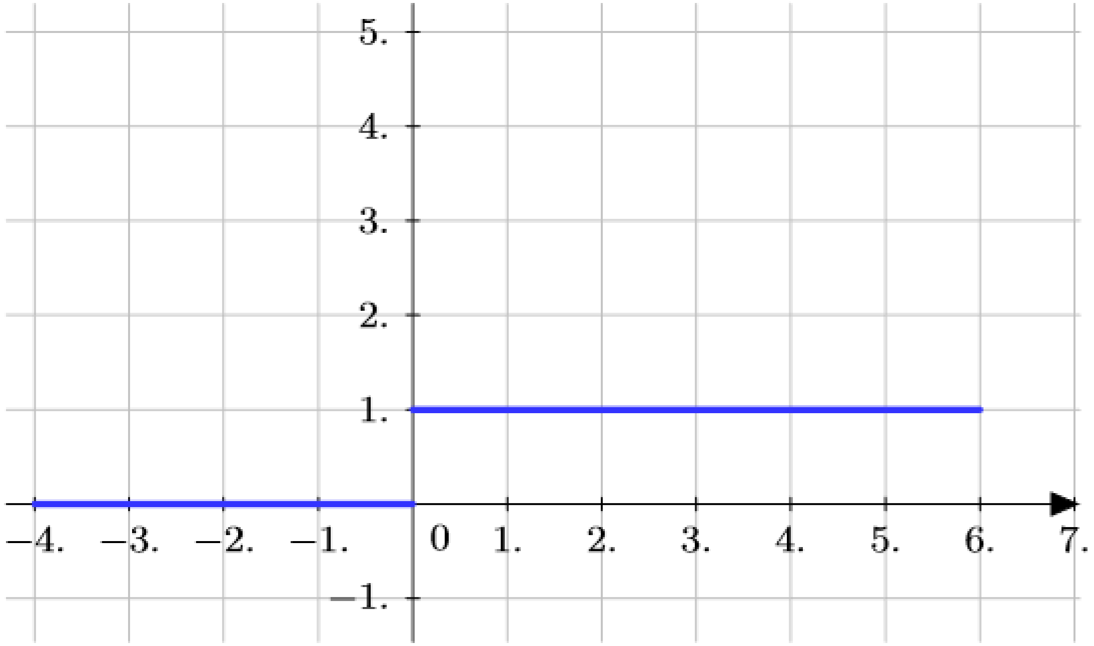


- Does it have values at $x = 0$?
- Does it have limits at $x = 0$?
  - $\lim_{x \to 0^-} f(x) =$
  - $\lim_{x \to 0^+} f(x) =$
- Does it continuous at $x = 1$?

Example

- Consider step function at $x = 0$.

$$f(x) = \begin{cases} 1 & x \geq 0 \\ 0 & x < 0 \end{cases}$$

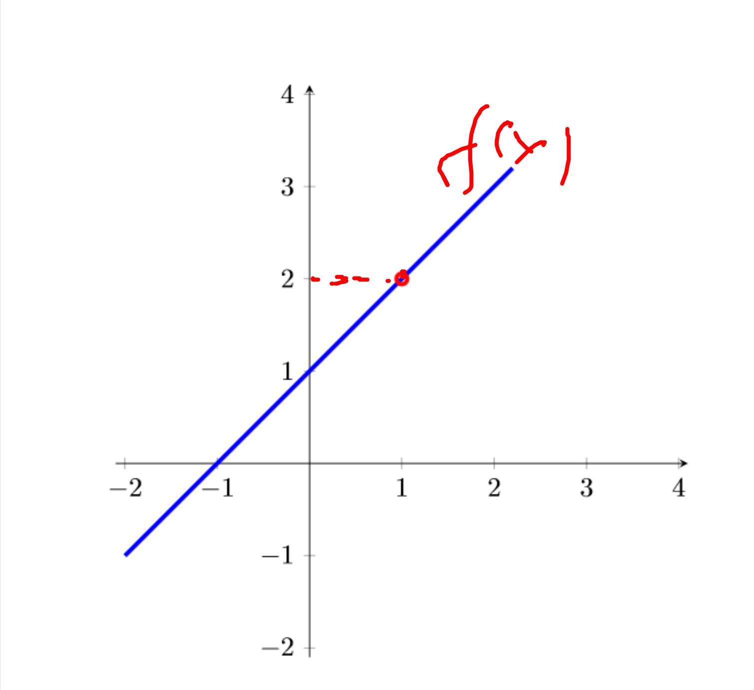


- Does it have values at $x = 0$?
- Does it have limits at $x = 0$?
  - $\lim_{x \to 0^-} f(x) =$
  - $\lim_{x \to 0^+} f(x) =$
- Does it continuous at $x = 1$?

Motivation: Numerical Approximation

- Let f be function

$$f(x) = \frac{x^2 - 1}{x - 1}$$



- At x = 1, we got a problem, $\frac{0}{0}$ is not defined.
- But looking in the neighbourhood we got the following tables.

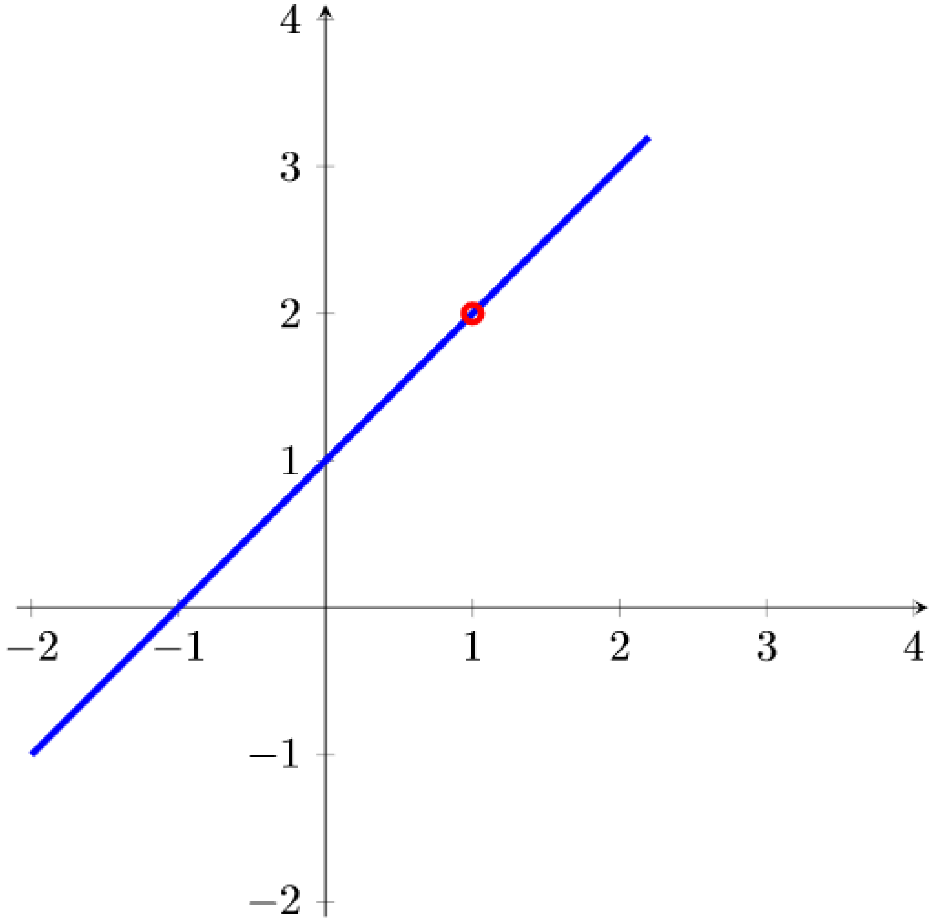
x	0.8	0.9	0.95	0.99	0.995	0.999	1	1.001	1.005	1.01	1.05	1.1	1.2
f(x)	1.8	1.9	1.95	1.99	1.995	1.999	??	2.001	2.005	2.01	2.05	2.1	2.2

Motivation: Numerical Approximation

- Let f be function

$$f(x) = \frac{x^2-1}{x-1}$$

- Does it have values at $x = 1$?
- Does it have limits at $x = 1$?
  - $\lim_{x \to 1^-} f(x) =$
  - $\lim_{x \to 1^+} f(x) =$
- Does it continuous at $x = 1$?

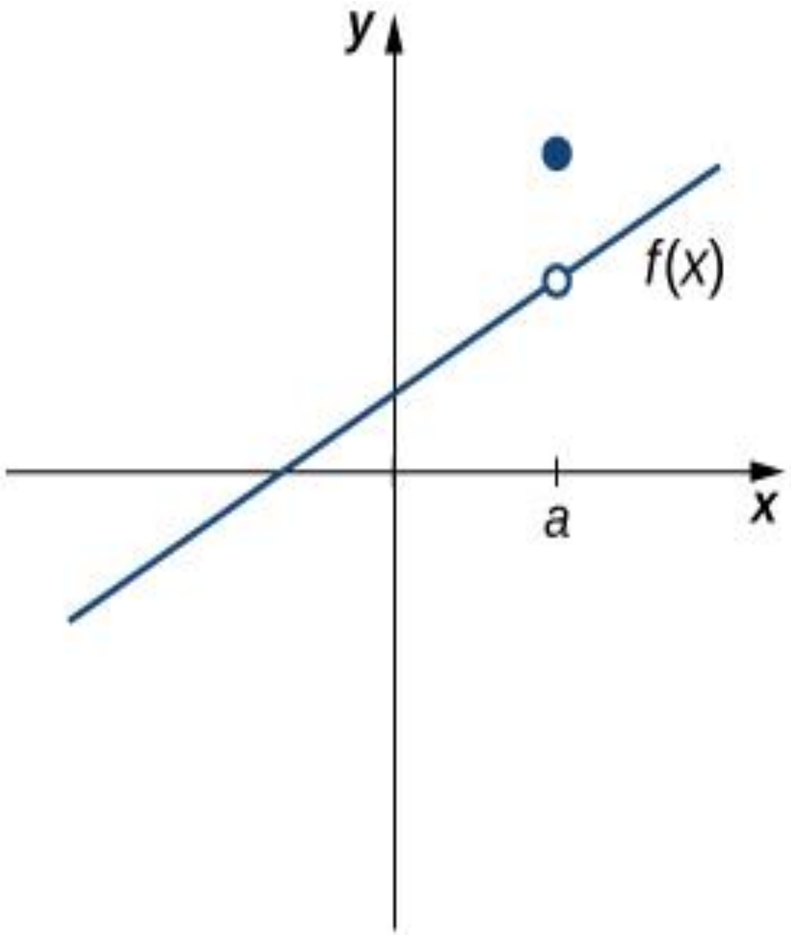


Motivation: Numerical Approximation

- Let f be function

$$f(x) = \begin{cases} \frac{x^2-(a+1)x+a}{x-a} & x \neq a \\ a+2 & x = a \end{cases}$$

- Does it have values at $x = a$?
- Does it have limits at $x = a$?
  - $\lim_{x \to a^-} f(x) =$
  - $\lim_{x \to a} f(x) =$
- Does it continuous at $x = a$?



Continuity on an interval

Limit

Continuity on an interval

Definition. Let I be an interval on real numbers. Let f be a function defined on I. Function f said to be *continuous on interval I*, if f continue at every point on I.

Continuity on an interval

Theorem. The following function are all continuous function in their respective domains.

- Polynomials,
- rational functions,
- root functions,
- trigonometric functions,
- inverse trigonometric functions,
- exponential function,
- logarithmic function,

Continuity on an interval

Theorem. Given two function f and g.

Let $\lim_{x \to a} g(x) = b$ and f continuous at b. Then we have

$$\lim_{x \to a} f(g(x)) = f(b).$$

In another word,

$$\lim_{x \to a} f(g(x)) = f\left(\lim_{x \to a} g(x)\right).$$

Continuity on an interval

Theorem. Given two function f and g and a real number a.

If f continuous at $g(a)$. Then the composition $f \circ g$ is continuous at a.

End of Session

Calculus 1